Derandomized Multi-block sign selection for PMEPR reduction of FBMC waveform

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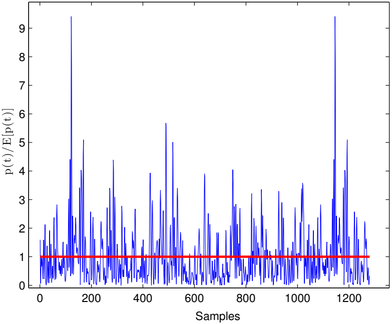
Outline

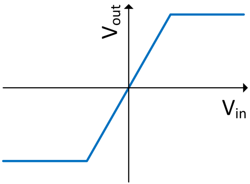
The PAPR problem

- High fluctuations of Multicarrier waveforms

$$s(n) = \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn}$$

- Nonlinearity of Power Amplifier (PA)
- In-band and out-of-band distortion



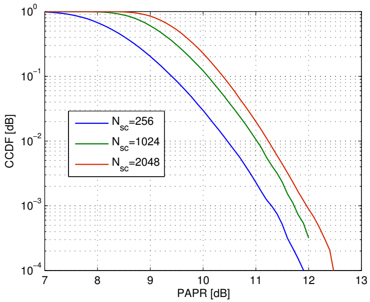


Definition and representation

- PMEPR for each symbol

$$\gamma = \frac{\max_{0 \le t < T} |s(t)|^2}{\mathrm{E}[|s(t)|^2]}$$

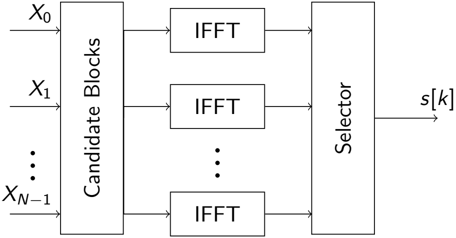
- Over intervals of length T, OFDM and OFDM/OQAM
- Complementary Cumulative Distribution Function (CCDF)



- N=256, 11.3 dB
- N=1024, 11.7 dB
- N=2048, 12 dB

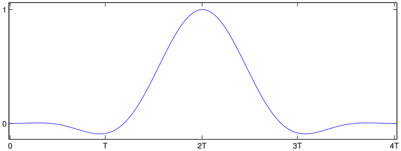
PAPR Reduction

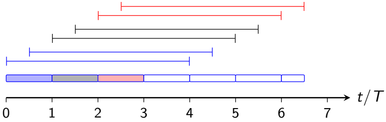
- Many methods since 90s.
- Selected Mapping (SLM)
- $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$
- $\mathbf{V}_m = [e^{j\phi_0}, e^{j\phi_1}, \ldots, e^{j\phi_{N-1}}] \quad 1 \leq m \leq M$
- Sign change



FBMC signal model I

- A multicarrier modulation
- Flexible pulse shape
- $TF = \frac{1}{2}$ for real data symbols
- Applied 2x faster
- $TF = 1$ for complex data symbols
- component length $(K + \frac{1}{2})LN$





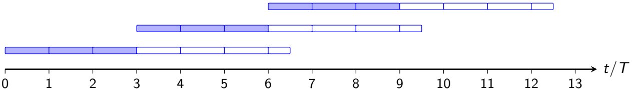
FBMC signal model II

- Continuous-time model:

$$s(t) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{N-1} a_{n,m} h(t - m\tau_0) e^{j\frac{2\pi}{T}nt} e^{j\phi_{n,m}}$$

- a segment $P = QLN$

$$s[p] = s^{(r)}[p] + \sum_{m=0}^{2Q-1} \sum_{n=0}^{N-1} \epsilon_{q(n,m)} a_{n,m} \gamma_{n,m}(p) \quad 0 \leq p < LQN$$



FBMC signal model III

- Focus:

$$s[p] = s^{(r)}[p] + \sum_{m=0}^{2Q-1} \sum_{n=0}^{N-1} \epsilon_{q(n,m)} a_{n,m} \gamma_{n,m}(p) \quad 0 \leq p < LQN$$

- $\max |s[p]| \leq \max |s_r[p]| + \max |s_i[p]|$

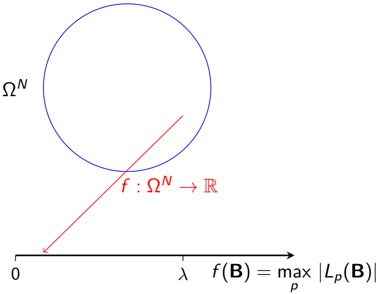
$$L_p(\epsilon) = L_p^r + \sum_{m=0}^{Q-1} \sum_{n=0}^{N-1} \epsilon_{q(n,m)} \Gamma_{n,m}(p) \quad 0 \leq p < 2P$$

- with $q(n,m) = (m-1)N + n$

$$L_p = L_p^r + \sum_{\text{all}} \begin{bmatrix} \epsilon_{1,1}\Gamma_{0,0}(p) & \epsilon_{1,2}\Gamma_{0,1}(p) & \cdots & \epsilon_{1,Q}\Gamma_{0,Q-1}(p) \\ \epsilon_{2,1}\Gamma_{1,0}(p) & \epsilon_{2,2}\Gamma_{1,1}(p) & \cdots & \epsilon_{2,Q}\Gamma_{1,Q-1}(p) \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{N,1}\Gamma_{N-1,0}(p) & \epsilon_{N,2}\Gamma_{N-1,1}(p) & \cdots & \epsilon_{N,Q}\Gamma_{N-1,Q-1}(p) \end{bmatrix}$$

Derandomized Algorithm I

- A random vector $\mathbf{B} \in \Omega^N$,
  Equiprobable, $\Omega = \{-1, 1\}$,
  $\mathbf{B} = [B_1, B_2, \ldots, B_N]^T$
- BAD event
  $A = \{\mathbf{B} : f(\mathbf{B}) \geq \lambda\}$
- $P(A) < 1 \rightarrow$ Exists B's with A not true



- Choose signs b_j iteratively such that

$$1 > P(A) \geq P(A|b_{1:j-1} = b^*_{1:j-1}) \geq P(A|b_{1:j} = b^*_{1:j}) \geq P(A|b_{1:N} = b^*_{1:N}) = 0$$

- Equiprobable RV $\rightarrow$

$$\begin{aligned} P(A|b_{1:j-1} = b^*_{1:j-1}) &= \sum_{\theta \in \Omega} P(\theta) P(A|b_{1:j-1} = b^*_{1:j-1}, b_j = \theta) \\ &\geq \min_{\theta \in \Omega} P(A|b_{1:j-1} = b^*_{1:j-1}, b_j = \theta) \end{aligned}$$

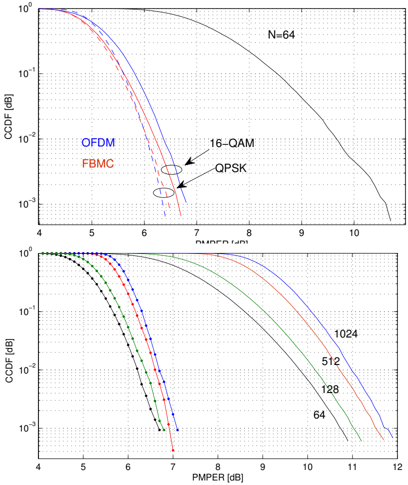
Derandomized Algorithm II

- Exhaustive search?!
- Upperbound $P(A|b_{1:j} = b^*_{1:j}) \leq F(b^*_{1:j})$
  - (i) $F < 1$
  - (ii) $F(b^*_{1:j-1}) \geq \min_{\theta \in \Omega} F(b^*_{1:j-1}, b_j = \theta)$
- In this problem:
  - Union bound:
    $P(\max_{\theta \in \{-1,1\}} |L_p(\boldsymbol{\epsilon})| > \lambda | \epsilon_{1:j} = \epsilon^*_{1:j}) \leq \sum_p P(|L_p(\boldsymbol{\epsilon})| > \lambda | \epsilon_{1:j} = \epsilon^*_{1:j})$
  - Chernoff bound: $P(X > \mu) = P(e^{tX} > e^{t\mu}) < \frac{\mathrm{E}[e^{tX}]}{e^{t\mu}}$
- Solution:
  - $F_p(\epsilon^*_{1:j}) = 2e^{-t\lambda} \cosh t(L_p(\epsilon^*_{1:j}) + L^r_p) \prod_{(n,m) \in \rho^c_j} \cosh t\Gamma_{n,m}(p)$
  - $\lambda = \sqrt{2(3QNE^o_{\max} h_{\max} + L^2_m) \ln 4P}$

Performance

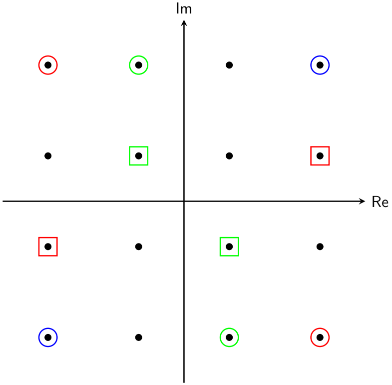
- Same for $Q = 1$ and higher.
- Slightly better for QPSK
- Almost the same for OFDM and FBMC
- PMPER grows faster for uncoded signal

N	at CCDF=10^{-3}
64	4 dB
128	4.2 dB
512	4.6 dB
1024	4.6 dB



Receiver

- Side Information or Rate Loss
- Detection by discarding signs



Conclusion

- High reduction performance.
- Works as well with FBMC waveform and overlappings
- Same performance as that of OFDM.

Thank you!